DATA 1010
In-class exercises
Samuel S. Watson
24 October 2018

Problem 1

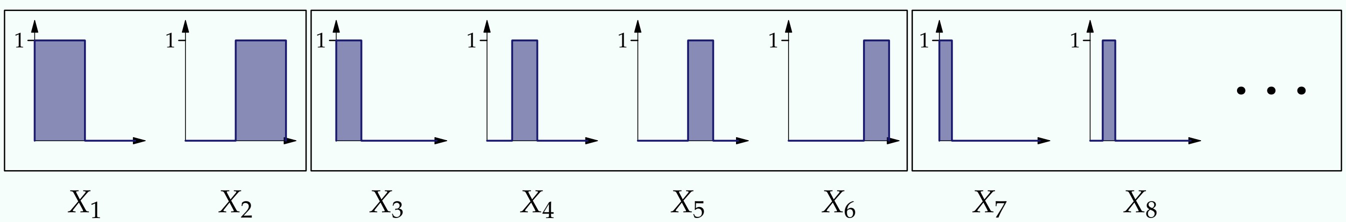
Chebyshev's inequality says that the probability that X deviates from its mean by more than k standard deviations is no greater than $1/k^2$.

The U.S. mint produces dimes with an average diameter of 0.5 inches and standard deviation 0.01. Using Chebyshev's inequality, give a lower bound for the number of coins in a lot of 400 coins that are expected to have a diameter between 0.48 and 0.52.

Problem 2

We say that a sequence of random variables $X_1, X_2, \ldots$ converges to a random variable X in probability if $\mathbb{P}(|X_n - X| > \epsilon) \to 0$ as $n \to \infty$, for any $\epsilon > 0$.

Consider the sequence of random variables defined on $\Omega = [0,1]$ (with probability measure $\mathbb{P}(A) = \text{length}(A)$) as follows:



In other words, the first two random variables are the indicator functions of the first half and the last half of the unit interval. The next four are the indicators of the first quarter, the second quarter, the third quarter, and the fourth quarter of the unit interval. The next eight are indicators of width-one-eighth intervals sweeping across the interval, and so on. For concreteness, suppose that each random variable is equal to 1 at any point ω where the random variable is discontinuous.

Show that $X_n \to 0$ in probability.

Problem 3

Define $f_n(x) = n\mathbf{1}_{0 \leq x \leq 1/n}$, and let ν_n be the probability measure with density f_n. Show that ν_n converges to the probability measure ν which puts of all its mass at the origin.

Problem 4

Use Chebyshev's inequality to show that if $X_1, X_2, \ldots$ is a sequence of independent samples from a finite-variance distribution ν, then the running average of the X_i's converges to the mean of ν.

Problem 5

Suppose we flip a coin which has probability 60% of turning up heads n times. Use the normal approximation to estimate the value of n such that the proportion of heads is between 59% and 61% with probability approximately 99%.