DATA 1010
In-class exercises
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Problem 1

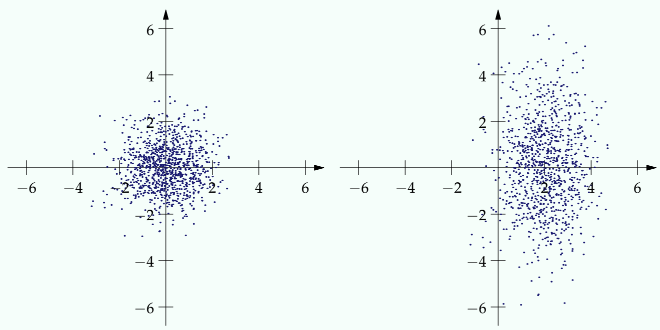
Suppose that Z has density $f_Z(z) = \frac{1}{\sqrt{2\pi}} e^{-z^2/2}$. Find the density of $X = \sigma Z + \mu$.

Problem 2

Suppose that $Z_1, \ldots, Z_n$ are independent standard normal random variables. Find the density of the random vector $\mathbf{Z} = [Z_1, \ldots, Z_n]$, and show that it is rotationally symmetric.

Problem 3

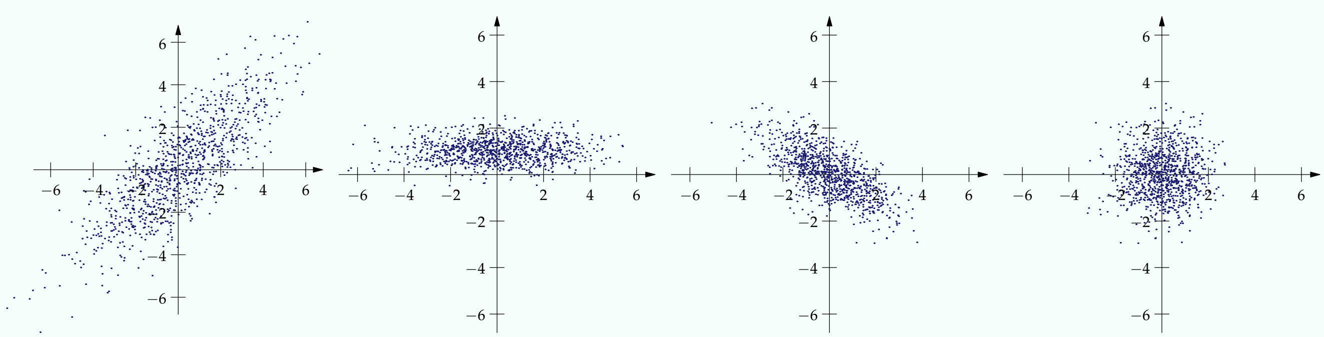
Find the function which maps the point cloud on the left to the point cloud on the right.



Problem 4

For each $i \in \{1, 2, 3\}$, the ith point cloud below is obtained by sampling 1000 times from $A_i Z + \mu_i$, where Z is a vector of two independent normal random variables, A_i is a 2×2 matrix of constants, and μ_i is a constant vector in $\mathbb{R}^2$. Approximate (A_i, μ_i) for each $i \in \{1, 2, 3\}$. (Note: A_i is not uniquely determined, so just find an A_i that works.)

For reference, a plot of 1000 independent samples from Z is shown in the fourth figure.



Problem 5

Find an expression for the probability density function of AZ, where Z is a vector of n independent standard normal random variables and A is an invertible $n \times n$ matrix.

Problem 6

Find the covariance matrix Σ of $A\mathbf{Z}$. Express the density of $A\mathbf{Z} + \mu$ in terms of Σ, where μ is a constant vector.