DATA 1010
In-class exercises
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Problem 1

The **geometric distribution** with parameter $p \in (0,1]$ is the distribution of the index of the first success in a sequence of independent Bernoulli trials.

Find the probability mass function of the geometric distribution.

Problem 2

Use Monte Carlo to find the mean and variance of the geometric distribution with parameter $p = 1/3$.

Problem 3

(i) Find the expected value of S, where S is a sum of 1000 independent Bernoulli random variables with success probability $p = \frac{3}{1000}$.

(ii) Find the probability mass function of S. Hint: find an expression representing the probability mass at each k from 0 to 1000, and then use Julia to evaluate it. You will need to define `n = big(1000)` and `p = big(3)/1000` because arbitrary precision arithmetic is required to avoid overflow issues.

(iii) Compare your results to the probability mass function $m(k) = \frac{3^k}{k!}e^{-3}$ defined on $\{0,1,2,\ldots\}$.

Problem 4

Suppose $\lambda > 0$, and find the mean and variance of a sum of n independent Bernoulli random variables with parameter $p = \lambda/n$ (where $n > \lambda$). Use your results to posit values for the expectation and variance of a Poisson random variable with parameter λ.

Problem 5

Suppose that the number of typos on a page is a Poisson random variable with mean $\lambda = \frac{1}{3}$.

(i) Provide an explanation for why the Poisson distribution might be a good approximation for the distribution of typos on a page.

(ii) Find the probability that a particular page is typo-free.

Problem 6

Imagine placing a light bulbs activated by independent Bernoulli(λ/n) random variables at every multiple of $1/n$ on the positive real number line. Consider the position X_n of the **leftmost lit bulb**.

(i) For each $t > 0$, find the limit as $n \to \infty$ of $\mathbb{P}(X_n > t)$.

(ii) Find the PDF associated with the measure that you found in part (a).