DATA 1010 In-class exercises Samuel S. Watson 15 October 2018

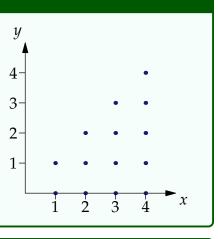
Problem 1

Consider a pair of random variables (X, Y) whose joint distribution is supported on $[0, 1]^2$ with density $6x^2y$. Show that *X* and *Y* are independent.

Now suppose the joint density is $\frac{3}{2}(x^2 + y^2)$. Show that *X* and *Y* are not independent.

Problem 2

Consider a pair of random variables *X* and *Y* with joint distribution *m*, where *m* is the probability mass function shown. Find the conditional distribution of *Y* given X = x for each value of *x*.



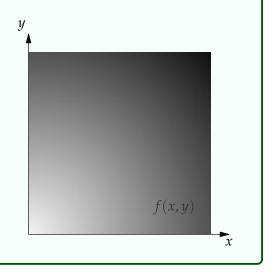
Problem 3

Suppose that *f* is the function which returns 2 for any point in the triangle with vertices (0,0), (1,0), and (0,1) and otherwise returns 0. If (X, Y) has joint PDF *f*, find the conditional expectation of *Y* given $\{X = x\}$.

Problem 4

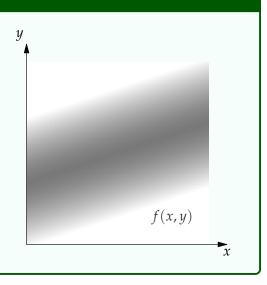
Given that *X* and *Y* have joint PDF $f(x, y) = \frac{3}{2}(x^2 + y^2)$ on $[0, 1]^2$, find the conditional expectation of *Y* given *X*.

Begin by sketching an estimate of the conditional expectation on the graph shown.



Problem 5

Given that *X* and *Y* have joint PDF shown in the figure, sketch an estimate of the conditional expectation of *Y* given X = x.



Problem 6

Given that *X* and *Y* have joint PDF $f(x, y) = \frac{9}{5}(1 - \sqrt{xy})$ on $[0, 1]^2$, find the conditional expectation of *Y* given *X*.

Begin by sketching an estimate of the conditional expectation on the graph shown.

