DATA 1010
In-class exercises
Samuel S. Watson
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## Problem 1

Consider the random variable $1_E$ which maps each $\omega \in E$ to 1 and each $\omega \in E^c$ to 0. Find the expected value of $1_E$.

## Problem 2

The expectation of a random variable need not be finite or even well-defined. Show that the expectation of the random variable which assigns a probability mass of $2^{-n}$ to the point $2^n$ (for all $n \geq 1$) is not finite.

Consider a random variable $X$ whose distribution assigns a probability mass of $2^{-|n|-1}$ to each point $2^n$ for $n \geq 1$ and a probability mass of $2^{-|n|-1}$ to $-2^n$ for each $n \leq -1$. Show that $\mathbb{E}[X]$ is not well-defined. (Note: a sum $\sum_{x\in\mathbb{R}} f(x)$ is not defined if $\sum_{x\in\mathbb{R}\,:\,f(x)>0} f(x)$ and $\sum_{x\in\mathbb{R}\,:\,f(x)<0} f(x)$ are equal to $\infty$ and $-\infty$, respectively.)

## Problem 3

Shuffle a standard 52-card deck, and let $X$ be the number of consecutive pairs of cards in the deck which are both red. Find $E[X]$.

Write some code to simulate this experiment and confirm that your answer is correct. Hint: store the deck of undrawn cards as a `Set`, and `pop!` cards from it as you draw. You can draw a random element from a set `S` using `rand(S)`.

## Problem 4

Show that variance satisfies the properties

$$\begin{cases} \operatorname{Var}(aX) = a^2 \operatorname{Var} X, & \text{for all random variables } X \text{ and real numbers } a \\ \operatorname{Var}(X+Y) = \operatorname{Var}(X) + \operatorname{Var}(Y), & \text{if } X \text{ and } Y \text{ are independent random variables} \end{cases}$$