

DATA 1010
IN-CLASS EXERCISES
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Problem 1

Consider the random variable 1_E which maps each $\omega \in E$ to 1 and each $\omega \in E^c$ to 0. Find the expected value of 1_E .

Problem 2

Find the expected value of $X + Y$, where X and Y are independent random variables whose distributions have constant probability mass functions on $\{0, 1, 2, 3\}$. What is the relationship between $\mathbb{E}[X + Y]$, $\mathbb{E}[X]$, and $\mathbb{E}[Y]$?

Problem 3

Consider a uniformly random permutation of the integers $\{1, 2, 3, 4\}$. Let X be 3 if the first two numbers are 1 and 2, and let it be 2 otherwise. Let Y be 6 if the second and third numbers are 2 and 3; let it be 4 if the second number is 2 and the third number is not 3; and let it be 0 if the second number is not 2.

Find $\mathbb{E}[X + Y]$, $\mathbb{E}[X]$, and $\mathbb{E}[Y]$.

Problem 4

Consider two unfair coins: the first has probability $\frac{3}{5}$ of coming up heads, and the second has probability $\frac{1}{3}$ of coming up heads. We define X to be 2 if the first coin is heads and -1 otherwise, and we define Y to be 1 if the first coin is tails and the second coin is heads, and 2 otherwise.

Show that $\mathbb{E}[X + Y] = \mathbb{E}[X] + \mathbb{E}[Y]$.

Problem 5

Suppose a fair coin is flipped and a fair die is rolled. We let X be 1 or 2 if the coin turns up tails or heads, respectively, and we let Y be the value of the die roll.

(a) Calculate $\mathbb{E}[XY]$ as well as $\mathbb{E}[X]$ and $\mathbb{E}[Y]$. What is the relationship between these three?

(b) Suppose that $Z = X + Y$ and calculate $\mathbb{E}[Z]$ and $\mathbb{E}[ZY]$. Do $\mathbb{E}[ZY]$, $\mathbb{E}[Z]$, and $\mathbb{E}[Y]$ have the same relationship you found in (a)?

Problem 6

Shuffle a standard 52-card deck, and let X be the number of consecutive pairs of cards in the deck which are both red. Find $\mathbb{E}[X]$.

Write some code to simulate this experiment and confirm that your answer is correct. Hint: store the deck of undrawn cards as a `Set`, and `pop!` cards from it as you draw. You can draw a random element from a set `S` using `rand(S)`.