DATA 1010
In-class exercises
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Problem 1

A problem on a test requires students to match molecule diagrams to their appropriate labels. Suppose there are three labels and three diagrams and that a student guesses a matching uniformly at random. Let X denote the number of diagrams the student correctly labels.

(a) What is the probability mass function of the conditional distribution of X given the event $X \geq 1$?

(b) What is the probability mass function of the conditional distribution of X given the event that the student knows the first matching and has to guess at the other two?

Problem 2

Consider the following experiment: we roll a die, and if it shows 2 or less we select Urn A, and otherwise we select Urn B. Next, we draw a ball uniformly at random from the selected urn. Urn A contains one red and one blue ball, while urn B contains 3 blue balls and one red ball.

Find a probability space Ω which models this experiment, find a pair of events E and F such that $\mathbb{P}(E \mid F) = \frac{3}{4}$.

Problem 3

Find the maximum possible value of $\frac{|A\mathbf{x}|}{|\mathbf{x}|}$ where $\mathbf{x} \in \mathbb{R}^3$ and

$$A = \begin{bmatrix} 4 & 11 & 14 \\ 8 & 7 & -2 \end{bmatrix}.$$

Problem 4

Express the largest representable `Float64` in base-10 scientific notation, accurate to 3 decimal places. Express the smallest positive representable Float64 in base-10 scientific notation, accurate to 3 decimal places.

Problem 5

Suppose that A is a matrix with the property that each column has norm 3 and every pair of distinct columns has angle 60 degrees between them. Find $A'A$.

Problem 6

Find $\lim_{n\to\infty} \begin{bmatrix} 81 & 80 & -440 \\ -20 & -19 & 110 \\ 11 & 11 & -\frac{119}{2} \end{bmatrix}^n$, given the diagonalization

$$\begin{bmatrix} 81 & 80 & -440 \\ -20 & -19 & 110 \\ 11 & 11 & -\frac{119}{2} \end{bmatrix} = \begin{bmatrix} -15 & -16 & 80 \\ 4 & 5 & -20 \\ -2 & -2 & 11 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \frac{1}{2} \end{bmatrix} \begin{bmatrix} -15 & -16 & 80 \\ 4 & 5 & -20 \\ -2 & -2 & 11 \end{bmatrix},$$