DATA 1010
In-class exercises
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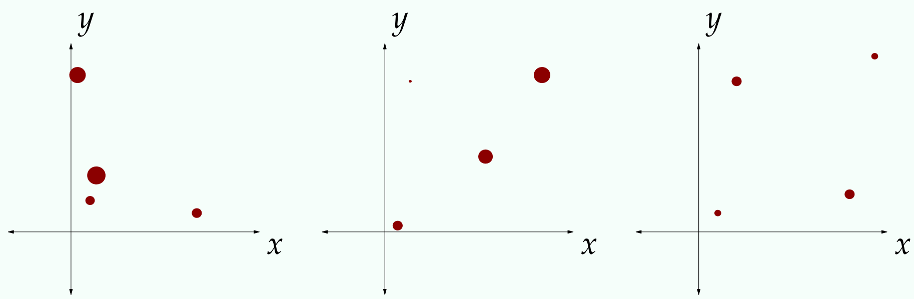
Problem 1

Suppose that X and Y are independent random variables whose distributions have constant probability mass functions on $\{0, 1, 2, 3\}$. Make a spike graph for the probability mass function of $X + Y$.

Problem 2

The first figure below shows the probability mass function for the joint distribution of two random variables X_1 and Y_1. The second and third figures show the joint distributions of (X_2, Y_2) and (X_3, Y_3).

For which value of i is $\mathbb{P}(Y_i > X_i)$ the largest?



Problem 3

Suppose that X is a random variable whose distribution has PMF $m_X(1) = 1/5$, $m_X(7) = 1/5$, and $m_X(\sqrt{3}) = 3/5$.

Suppose that Y is a random variable whose distribution has PMF $m_Y(1) = 1/4$, $m_Y(3) = 1/4$, $m_Y(11.5) = 1/4$, and $m_Y(-4) = 1/4$.

Suppose that X and Y are independent, and call their joint PMF $m_{(X,Y)}$. For how many ordered pairs (x, y) do we have $m_{(X,Y)}(x, y) > 0$?

Problem 4

Show that if E and F are independent, then E and F^c are also independent.

Problem 5

The 52 cards in a standard deck are shuffled and dealt out in four hands of 13 cards each. What is the conditional probability, given that the first two hands contain 8 of the 13 spades, that the fourth hand contains exactly 3 of the remaining spades?

Problem 6

A problem on a test requires students to match molecule diagrams to their appropriate labels. Suppose there are three labels and three diagrams and that a student guesses a matching uniformly at random. Let X denote the number of diagrams the student correctly labels.

(a) What is the probability mass function of the conditional distribution of X given the event $X \geq 1$?

(b) What is the probability mass function of the conditional distribution of X given the event that the student knows exactly one of the matchings and has to guess at the other two?

Problem 7

Consider the following experiment: we roll a die, and if it shows 2 or less we select Urn A, and otherwise we select Urn B. Next, we draw a ball uniformly at random from the selected urn. Urn A contais one red and one blue ball, while urn B contains 3 blue balls and one red ball.

Find a probability space Ω which models this experiment, find a pair of events E and F such that $\mathbb{P}(E \mid F) = \frac{3}{4}$.

Problem 8

Consider the random variable 1_E which maps each $\omega \in E$ to 1 and each $\omega \in E^{\mathsf{c}}$ to 0. Find the expected value of 1_E.

Problem 9

Find the expected value of $X + Y$, where X and Y are independent random variables whose distributions have constant probability mass functions on $\{0, 1, 2, 3\}$.