DATA 1010
In-class exercises
Samuel S. Watson
28 September 2018

Problem 1

Write a program to play the five-dice game many times, and determine the proportion of two-pair rolls you get.

Solution

```
"""
Roll five six-sided dice T times and determine
the number of two-pair results
"""
function play(T)
   sum(twopair(rand(1:6,5)) for i=1:T)
end
T = 1000
play(T)/T
```

Problem 2

Suppose that X is a random variable with CDF F_X and that $Y = X^2$. Express $\mathbb{P}(Y > 9)$ in terms of the function F_X. For simplicity, assume that $\mathbb{P}(X = -3) = 0$.

(a) $1 - F_X(3) + F_X(3)$

(b) $F_X(-3)$

(c) $F_X(3)$

(d) $F_X(3) + F_X(-3)$

Solution

By definition of Y, we have that $Y^2 > 9$ if $X < -3$ or $X > 3$. Since these events are mutually exclusive, we have

$$\begin{aligned}\mathbb{P}(Y > 9) &= \mathbb{P}(X < -3) + \mathbb{P}(X > 3)\\ &= \mathbb{P}(X < -3) + 1 - \mathbb{P}(X \le 3)\\ &= F_X(-3) + 1 - F_X(3),\end{aligned}$$

where the last step follows since $\mathbb{P}(X < -3) = \mathbb{P}(X \le 3)$ for this random variable X.

Problem 3

Random variables with the same cumulative distribution function are not necessarily equal as random variables, because the probability mass sitting at each point on the real line can come from different ω's.

For example, consider the two-fair-coin-flip experiment and let X be the number of heads. Find another random variable Y which is not equal to X but which has the same distribution as X.

Solution

If we define Y to be the number of *tails*, then it's clear from symmetry that it has the same distribution as X. Furthermore, X and Y are unequal as random variables because if $X = 0$, then $Y = 2$ (and vice versa).

(In fact, we can express Y in terms of X as $Y = 2 - X$.)

Problem 4

Consider a computer program which rolls two virtual dice and returns roll results with probabilities shown in the table.

What is the probability that Die 1 shows 4?

Die 1

Die 2	1	2	3	4	5	6
1	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{3}{36}$	$\frac{1}{36}$
2	$\frac{1}{72}$	$\frac{1}{36}$	$\frac{1}{72}$	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{1}{36}$
3	$\frac{1}{36}$	$\frac{1}{72}$	$\frac{1}{72}$	$\frac{1}{72}$	$\frac{1}{36}$	$\frac{2}{36}$
4	$\frac{1}{72}$	$\frac{1}{72}$	$\frac{1}{72}$	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{1}{36}$
5	$\frac{2}{36}$	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{1}{36}$
6	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{1}{72}$	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{1}{72}$

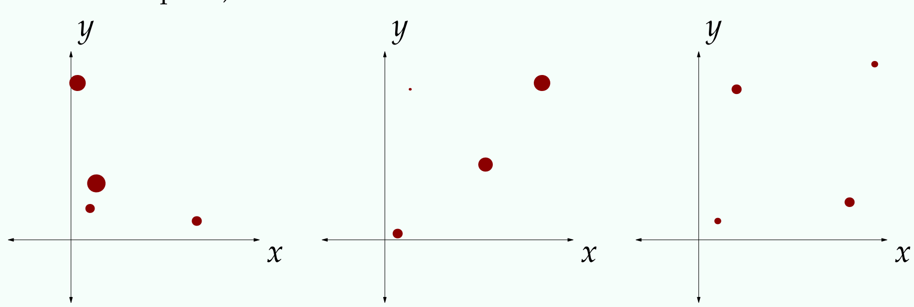
Solution

The event that the first die shows 4 can be written as a disjoint union of the events $\{\text{Die } 1 = 4\} \cap \text{Die } 2 = j$ where j ranges over the integers 1 to 6. We get

$$\begin{aligned} \mathbb{P}(\text{Die } 1 = 4) &= \sum_{j=1}^{6} \mathbb{P}(\text{Die } 1 = 4,\ \text{Die } 2 = j) \\ &= \frac{1}{36} + \frac{1}{36} + \frac{1}{72} + \frac{1}{36} + \frac{1}{36} + \frac{1}{36} \\ &= \frac{11}{72}. \end{aligned}$$

Problem 5

Determine which of the following joint distributions on (X, Y) has the property that each random variable X and Y has the same marginal distribution. (Note: each disk indicates a probability mass at a point, with the size of the disk proportional to the mass at that point)



Solution

We find the distribution of X by summing the joint distribution along vertical lines, and we obtain the distribution of Y by summing along horizontal lines. Only for the third distribution do these two procedures give the same results.