DATA 1010
In-class exercises
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Problem 1

Let $\Omega = \{(\mathtt{H},\mathtt{H}), (\mathtt{H},\mathtt{T}), (\mathtt{T},\mathtt{H}), (\mathtt{T},\mathtt{T})\}$, and let $m(\omega) = \frac{1}{4}$ for each $\omega \in \Omega$.
Identify a mathematical object in the model (Ω, m) which can be said to correspond to the phrase "the first flip turns up heads". Which of the following is true of this object?

(i) It is one of the values of the function m

(ii) It is the set Ω

(iii) It is a subset of Ω

(iv) It is one of the elements of Ω

Problem 2

Explain how to obtain the probability of an event from the probability mass function.
For concreteness, consider $\Omega = \{(\mathtt{H},\mathtt{H}), (\mathtt{H},\mathtt{T}), (\mathtt{T},\mathtt{H}), (\mathtt{T},\mathtt{T})\}$, a probability mass function which assigns mass $\frac{1}{4}$ to each outcome, and the event $\{(\mathtt{H},\mathtt{H}), (\mathtt{H},\mathtt{T})\}$.

Problem 3

Match each term to its corresponding set-theoretic operation. Assume that E and F are events.
For concreteness, you can think about the events "first flip comes up heads" and "second flip comes up heads" for the two-flip probability space we've been considering.

(a) the event that E and F both occur
(b) the event that E does not occur
(c) the event that either E occurs or F occurs

(i) the intersection $E \cap F$
(ii) the union $E \cup F$
(iii) the complement E^{c}

Problem 4

Suppose a group of n friends enter the lottery. For $i \in \{1, \ldots, n\}$ let E_i be the event that the ith friend wins. Express the following events using set notation.

1. At least one friend loses.
2. All friends win.
3. At least one friend wins.

Problem 5

What is the cardinality of the domain of the function $\mathbb{P}$ if

$$\Omega = \{(\mathtt{H},\mathtt{H}), (\mathtt{H},\mathtt{T}), (\mathtt{T},\mathtt{H}), (\mathtt{T},\mathtt{T})\}?$$

Problem 6

Consider events A and B where the occurrence of A implies the occurrence of B. For example, suppose A is the event that the Red Sox outscore the Yankees by 5 runs or more, and let B be the event that the Red Sox win the game. Which of the following is the set-theoretic relationship between the events A and B?

(a) $A \cap B = \emptyset$

(b) $A \subset B$

(c) $B \subset A$

Problem 7

Use the additivity property and the fact that $A = (A \cap B) \cup (A \cap B^c)$ to show that if $B \subset A \subset \Omega$, then $\mathbb{P}(B) \leq \mathbb{P}(A)$.

Problem 8

Show that $\mathbb{P}(A \cup B) \leq \mathbb{P}(A) + \mathbb{P}(B)$ for all events A and B.

Use this property to show that if A occurs with probability zero and B occurs with probability zero, then the probability that A *or* B occurs is also zero.