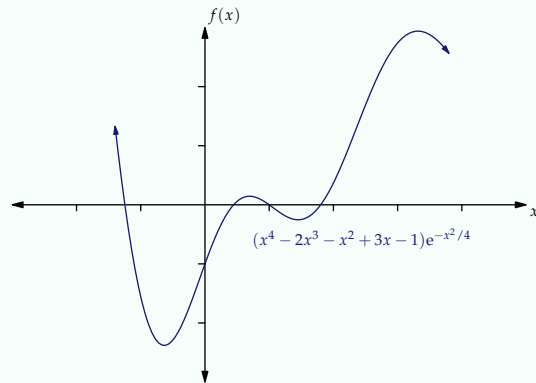


DATA 1010
IN-CLASS EXERCISES
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Problem 1

Consider the function $f(x) = (x^4 - 2x^3 - x^2 + 3x - 1)e^{-x^2/4}$. Implement the gradient descent algorithm for finding the minimum of this function.

- (i) If the learning rate is $\epsilon = 0.1$, which values of x_0 have the property that $f(x_n)$ is close to the global minimum of f when n is large?
- (ii) Is there a starting value x_0 between -2 and -1 and a learning rate ϵ such that the gradient descent algorithm does not reach the global minimum of f ? Use the graph for intuition.



Problem 2

Which of the following two lines of Julia code returns the larger value? You may take it as given that they do not return the same value. Explain.

```
sum(sqrt(k)^2 == k for k=1:100)
sum(sqrt(k^2) == k for k=1:100)
```

Problem 3

Suppose that $f : [0, 1] \rightarrow \mathbb{R}$ is a strictly increasing, continuous function such that $f(0) < 0 < f(1)$. The intermediate value theorem tells us that $f(x_0) = 0$ for exactly one value of x_0 between 0 and 1.

Consider the following method for approximating x_0 :

- (i) Check the sign of $f(1/2)$.
- (ii) Depending on the result of (i), check the sign of either $f(1/4)$ or $f(3/4)$.
- (iii) Depending on the results of (i) and (ii), check the sign of $f(1/8)$ or $f(3/8)$ or $f(5/8)$ or $f(7/8)$.
- (iv) Continue in this way, repeatedly narrowing down the interval which contains x_0 , for some fixed number of iterations.

Answer the following questions about this algorithm.

- (a) What would be the maximum possible number of iterations required to determine which two `Float64` values x_0 lies between?
- (b) Implement this algorithm in Julia. Your function should pass the following tests.

```
findzero(x->x^2-1/2) == 1/sqrt(2)
```

```
findzero(x->x-1/2) == 1/2
```

Problem 4

The matrix $\begin{bmatrix} 1/2 & \sqrt{3}/2 \\ -\sqrt{3}/2 & 1/2 \end{bmatrix}$ represents an n° rotation about the origin.

- (a) Check that the determinant of this matrix is compatible with the claim that it represents a rotation.
- (b) Find n .