DATA 1010
In-class exercises
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Problem 1

Consider the sequence $\{\text{mod}(3 \cdot 2^n, 11)\}_{n=1}^{100}$. Use Julia to show that each number from 1 to 10 appears exactly 10 times in this sequence. Also, use Julia to show that a_{2k} is smaller than a_{2k-1} for far more than half the values of k from 1 to 50. Hint: `countmap(a)` tells you how many times each element in the collection `a` appears. To use this function, do `using StatsBase` first.

Repeat these tests on the sequence whose kth term is the kth digit in the decimal representation of π: `reverse(digits(floor(BigInt,big(10)^99*pi)))`.

Problem 2

Use difference quotients to approximate the derivative of $f(x) = x^2$ at $x = \frac{2}{3}$, with $\epsilon = 2^k$ as k ranges from -60 to -20. What is the least error over these values of k? How does that error compare to machine epsilon?

Problem 3

In this exercise, we will explain why

$$f\left(\begin{bmatrix} x & 1 \\ 0 & x \end{bmatrix}\right) = \begin{bmatrix} f(x) & f'(x) \\ 0 & f(x) \end{bmatrix}, \tag{3.1}$$

for any polynomial f.

(i) Check that (3.1) holds for the identity function (the function which returns its input) and for the function which returns the multiplicative identity.

(ii) Check that if (3.1) holds for two differentiable functions f and g, then it holds for the sum $f + g$ and the product fg.

(iii) Explain why (3.1) holds for any polynomial function $f(x)$.

Problem 4

Use automatic differentiation to find the derivative of $f(x) = (x^4 - 2x^3 - x^2 + 3x - 1)e^{-x^4/4}$ at the point $x = 2$. Compare your answer to the true value of $f'(2)$.

Hint: You'll want to define f using

```
using LinearAlgebra
f(t) = exp(-t^2/4)*(t^4 - 2t^3 - t^2 + 3t - I)
```

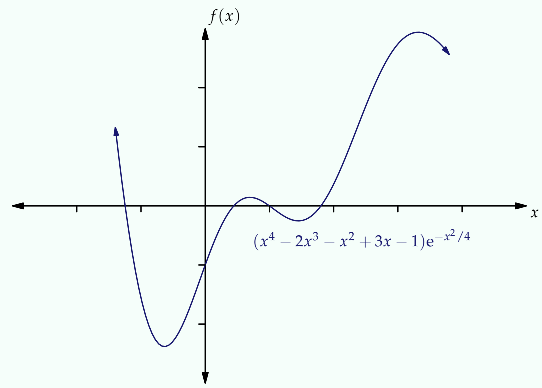
where `I` is an object which is defined to behave like multiplicative identity (note that subtracting the identity matrix is the appropriate matrix analogue of subtracting 1 from a real number).

Also, to help check your answer, here's the symbolic derivative of f:

```
df(t) = (-t^5 + 2*t^4 + 9*t^3 - 15*t^2 - 3*t + 6)*exp(-t^2/4)/2
```

Problem 5

Consider the function $f(x) = (x^4 - 2x^3 - x^2 + 3x - 1)e^{-x^4/4}$. Implement the gradient descent algorithm for finding the minimum of this function.



(i) If the learning rate is $\epsilon = 0.1$, which values of x_0 have the property that $f(x_n)$ is close to the global minimum of f when n is large?

(ii) Is there a starting value x_0 between -2 and -1 and a learning rate ϵ such that the gradient descent algorithm does not reach the global minimum of f? Use the graph for intuition.