DATA 1010
In-class exercises
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Problem 1

Find the difference between the largest and second-largest representable `Float64` values.

Problem 2

Between which two consecutive powers of 2 are the representable numbers exactly the integers in that range?

Problem 3

Discuss the error in each of the following scenarios using the terms *roundoff error, truncation error, or statistical error*.

(i) We use the trapezoid rule with 1000 trapezoids to approximate $\int_0^{10} \frac{1}{4+x^4}\,\mathrm{d}x$.

(ii) We are trying to approximate $f'(5)$ for some function `f` that we can compute, and we attempt to do so by running `(f(5 + 0.5^100) - f(5))/0.5^100`. We fail to get a reasonable answer.

(iii) To approximate the minimum of a function $f : [0,1] \to \mathbb{R}$, we evaluate f at 100 randomly selected points in $[0,1]$ and return the smallest value obtained.

Problem 4

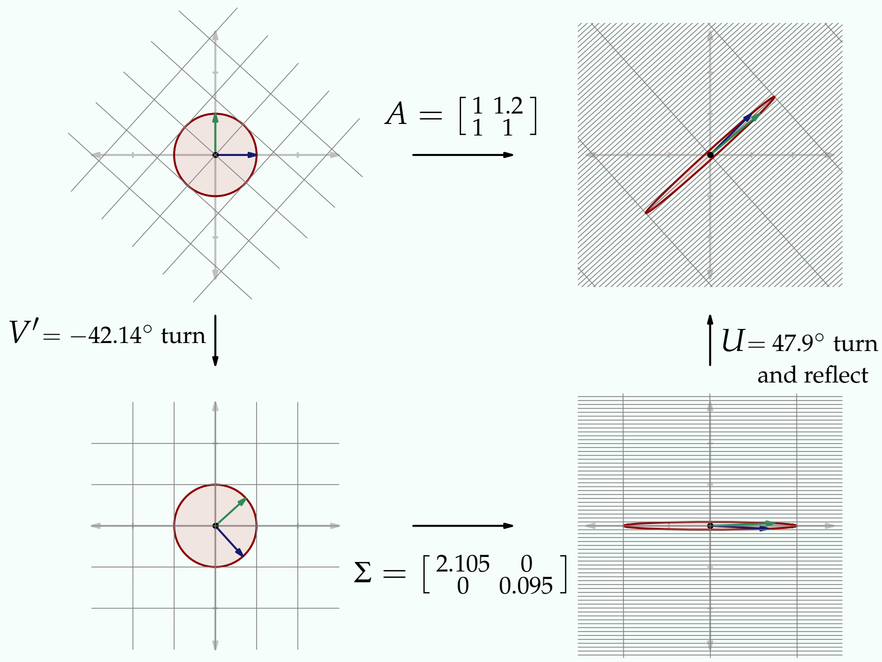
Consider a function $S : \mathbb{R} \to \mathbb{R}$. If the input changes from a to $a + \Delta a$ for some small value Δa, then the output changes to approximately $S(a) + \frac{\mathrm{d}}{\mathrm{d}a}S(a)\,\Delta a$. Calculate the ratio of the *relative change* in the output to the relative change in the input, and show that you get

$$\frac{a\frac{\mathrm{d}}{\mathrm{d}a}S(a)}{S(a)}.$$

Problem 5

Show that the condition number of a matrix A is equal to the ratio of its largest and smallest singular values.

Interpret your resulting by explaining how to choose two vectors with small relative difference which are mapped to two vectors with large relative difference by A, assuming that A has a singular value which is many times larger than another. Use the figure below to help with the intuition.



Problem 6

The determinant of a 2×2 matrix can be close to zero either because both singular values are small or because one of the singular values is very small while the other is not.

Consider a matrix like

$$\begin{bmatrix} 1.01 & 1 \\ 1 & 1 \end{bmatrix}$$

This matrix has a small determinant (0.01). Are both singular values small? Is it possible that a 2×2 matrix has unit-length columns and two small singular values (having length less than or equal to 0.1, say)?