DATA 1010
In-class exercises
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Problem 1

In a first linear algebra course, one often learns to find the null space of a matrix using by-hand row reduction to solve the corresponding linear system. In Julia, you can find the null space of a matrix using `nullspace`. Try some small matrices (including some with nontrivial null spaces) and check that this function behaves as expected.

In a first linear algebra course, one often learns to find a basis for span of the columns of a matrix using by-hand row reduction to solve a linear system. In Julia, you can find such a basis by selecting the first k columns from the U factor in the SVD, where k is the rank of the matrix (which can be calculated using `rank`). Try some small matrices and check that this algorithm behaves as expected.

Bonus: check out how Julia's standard library implements `nullspace`.

Problem 2

Consider the matrix `A = [3 4; 2 1; 1 3; 2 5; 6 3]` which encodes the locations of five points in the first quadrant (each row contains the coordinates of a point).

In this problem we will find the line ℓ through the origin which minimizes the sum of the squares of the distances from the five points to ℓ.

(a) Draw the points and make your best guess at the optimal line ℓ (just eyeball it).

(b) Consider the vector $\mathbf{u}$ which extends from the origin to the point on ℓ which is in the first quadrant and is one unit from the origin. Show that the squared distance from any point $\mathbf{a} \in \mathbb{R}^2$ to ℓ is $|\mathbf{a}|^2 - (\mathbf{a} \cdot \mathbf{u})^2$.

(c) Show that the unit vector $\mathbf{u}$ which minimizes the sum of squared distances from the given points to ℓ is equal to the vector $\mathbf{u}$ which maximizes $|A\mathbf{u}|^2$.

(d) Find the value of $\mathbf{u}$ which maximizes $|A\mathbf{u}|^2$ by writing $\mathbf{u} = [\cos\theta, \sin\theta]$ and maximizing the single-variable function $|A[\cos\theta, \sin\theta]|^2$. To help you get started:

```julia
using Optim
f(θ) = ...
optimize(f,0,π/2) # _minimizes_ f over [0,π/2]
```

(e) How close was your guess from (a)?