DATA 1010
In-class exercises
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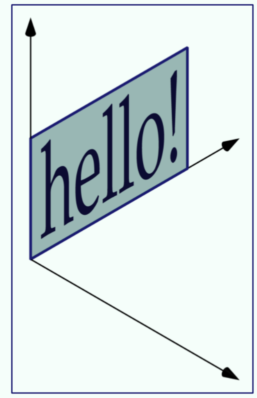
Problem 1

Describe how to transform the pixels in the image shown to get a obtain a *rectangular* image with the word “hello” on it.



Problem 2

Show that if T is an injective linear transformation and $\{\mathbf{v}_1, \ldots, \mathbf{v}_n\}$ is a linearly independent list of vectors, then $\{T(\mathbf{v}_1), \ldots, T(\mathbf{v}_n)\}$ is a linearly independent list.

Problem 3

Write down a small matrix equation of the form $A\mathbf{x} = \mathbf{b}$ which has a unique solution $\mathbf{x}$. Confirm that `A \ b` returns the correct solution.

Next, write a square matrix equation for which the matrix of coefficients is not invertible. Confirm that `A \ b` throws an error.

Problem 4

Consider a random symmetric $n \times n$ matrix A defined by

```
n = 10
A = zeros(n,n)
for i=1:n
    for j=1:i
        A[i,j] = rand()
        A[j,i] = A[i,j]
    end
end
```

and define $\mathbf{v}_0 = [1, 0, \ldots, 0] \in \mathbb{R}^n$. For $k \geq 0$, define $\mathbf{v}_{k+1} = \frac{A\mathbf{v}_k}{|A\mathbf{v}_k|}$. Then as $k \to \infty$, $\mathbf{v}_k$ converges to the eigenvector with the eigenvalue which is largest in absolute value.

Implement this algorithm in Julia and compare the eigenvector you find to the ones returned by Julia’s `eigen` function.

Problem 5

Explain why the algorithm in Problem 4 works. Are there any starting vectors $\mathbf{v}_0$ for which the algorithm would fail? Use your conclusions to explain how we might calculate the eigenvector whose eigenvalue is *second* largest in absolute value.